School mathematics for the 21st century

Some key influences

This paper contains background information on some factors that influence thinking about mathematics in schools for the 21st century. It is designed to be read in conjunction with the Discussion Paper entitled *School mathematics for the 21st century: What should school mathematics of the 21st Century be like?*, released by the Australian Association of Mathematics Teachers (AAMT) for National Mathematics Day 2009.

May 2009

Mathematics for the 'knowledge economy'

There is a strong argument that mathematics is increasingly important in our society. It is clear that the pervasive technologies of our times are, and will continue to be, substantially based on, and enablers of, mathematics. Those developing technologies need, of course, to be highly mathematically competent. On the broader level our society is very much driven by data and analyses of mathematical models that result from the use of the technologies. Everyone needs mathematical skills and capabilities.

On the other hand, however, these technologies effectively 'submerge' the popular perception of what constitutes 'mathematics' — mathematics seems nowhere near as important as it used to be. No-one needs to be able to manually do a whole range of things such as simple and now, very complex, calculation. These can be automated.

This paradox is resolved if we consider what we mean by mathematics. 'Low level' skills that can be more accurately and efficiently done by a machine are certainly much less relevant in use and can no longer be supported as the key outcomes of school mathematics *for their own sake*. The 21^{st} century requires mathematics of a higher order for citizens to be able to understand, work with and create mathematical models that are accessible and powerful in the context of current and emerging technologies. As a result, the important mathematics in schooling should be about this sort of mathematics.

That is not to say that formerly important 'lower level' skills are not important. In some, perhaps many cases they are, insofar as they are integral to being able to work with powerful mathematical tools, technologies and techniques. For example, the emergence of mental computation as an important component in young children's facility with number has clear practical uses. But mental computation is also important through its capacity to deepen understanding of how numbers 'work' and therefore critical to fluent use of mathematical models of all sorts.

There needs to be a major rethinking of school mathematics in the light of all this. Its purposes have to be in line with the mathematical needs of young people and their futures. School mathematics needs to maximise its fit with the expectations of the wide range of other stakeholders such as government, higher education, business and industry.

Hence mathematics content needs to be selected for clear reasons that link to the nature of the mathematics that citizens need in and from schooling, and into the future. These reasons do not include 'we have always done it.' Apparently heretical questions like "Should the kind of 'procedural' calculus that has been the pinnacle of achievement in school mathematics in the 20^{th} century remain so in the 21^{st}?" and "Does the emphasis on algebraic skills serve students' and the society's needs?" need to be debated.

Deep knowledge of mathematics is critical and this has implications for what happens in schools.

Methodologies for teaching, learning and assessment also need to be critically analysed to ensure that the ways of working and learning are consistent with building in young people the kind of view of mathematics — and themselves as mathematicians — that will support their success in the knowledge era, whatever their vocational pathway(s).

Another key consideration in reshaping school mathematics goes beyond the broad areas of 'content' and 'process'. It is the dimension of how students think of mathematics and come to 'know' mathematics. Gilbert and Macleod (2006)[1] discuss the concept of 'deep approaches to learning' that are characterised by learning as 'making sense of physical concepts and procedures – coherent and integrated understanding sought'; as 'seeing phenomena in the world in a new way – developing ability to apply knowledge and methods to see phenomena in new ways, mediated by a reflective dimension'; and as 'change as a person – awareness of being able to see the world differently makes them see themselves differently'. Higher-order metacognitive skills also need to be given explicit attention and developed alongside learning content and process in order for students to successfully adopt these 'deep approaches to learning'.

In relation to learning mathematics at university in particular, these authors cite Crawford, Gordon, Nicholas and Prosser (1998, p. 465)[2] who concluded that '…fragmented conceptions of mathematics are associated with surface approaches to learning mathematics... On the other hand, cohesive conceptions of mathematics are associated with deep approaches to learning mathematics…. students holding cohesive conceptions of mathematics adopt deep approaches to learning mathematics, and have very different interpretations of learning mathematics.' This is likely to be true, at an appropriate level of sophistication, for learners of mathematics in our schools as well.

Hence a core characteristic of school mathematics needs to be for students to have the capacity to build cohesive conceptions of mathematics and be 'deep' learners of mathematics. In simple terms it means that we need to limit the content in the curriculum so that students can become deep learners of mathematics.

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[1] Gilbert, R. & Macleod, H. 2006. *An analysis of the current suite of QSA Years 11 and 12 syllabuses: A report for the Queensland Studies Authority Review of Syllabuses for the Senior Phase of Learning*. Brisbane: Queensland Studies Authority.

[2] Crawford, K., Gordon, S., Nicholas, J. and Prosser, M. (1998) Qualitatively different experiences of learning mathematics at university, *Learning and Instruction,* 8, 5, 455-468.

Being numerate

'Numeracy' as a term has been extensively adopted by politicians, policy makers and educators over the past 10 years in particular. There needs to be clarity and a common sense of purpose for school mathematics and numeracy as distinct yet connected educational constructs in the 21st century.

A key starting point is the AAMT's current position on numeracy — *Policy on Numeracy Education in Schools* (AAMT; 1998)[3]. In that document the Association proposed a description of being numerate as using 'mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life'. This, in turn, requires the disposition and a confidence to use mathematics.

The description goes on to outline some different dimensions of 'being numerate'. These have been further developed, with the result that Hogan (2000)[4] summed up his work with Sue Willis, in particular, by identifying

> three types of know-how:
>
> > mathematical
> >
> > contextual
> >
> > strategic

that need to be 'blended' in order to be numerate in a context.

In relation to these 'know-hows', school mathematics in the 21st century needs to contribute to students' development of **mathematical** know-how — providing knowledge and conceptual understanding of important mathematics through 'deep' learning. School mathematics in the 21st century also needs to contribute to the development of students' **strategic** know-how — the orientation and capacity to think and work mathematically in the real world. Its contribution to students' **contextual** know-how is also related to students' experiences in mathematics of working mathematically in contexts.

[3] Australian Association of Mathematics Teachers Inc. 1998. *Policy on Numeracy Education in Schools*. Adelaide: Author.

[4] Hogan, J. 2000. 'Numeracy — across the curriculum?'. *The Australian Mathematics Teacher*, Vol. 56, No. 3, August. AAMT.

Generation Y and school mathematics

Some of the above discussion highlights one of the key characteristics of the students in schools today and over the next decade. It is that these young people expect learning to be meaningful and make sense to them. Recent attention to the views and characteristics of young people born in the 1990s (Generation Y) has identified some generalisations that need to be considered when thinking about school mathematics — these are the 'clients' after all.

A recent report from the Dusseldorp Skills Forum[5], indicates that today's young people aged 16-24 tend to have a strictly instrumentalist view of education: it is there to provide them with the skills and knowledge necessary to get a job. Nine of ten students interviewed said that courses are valuable to their future working lives and careers. Those who had gone on to get apprenticeships and vocational training or had gone straight into the workforce were inclined to speak dismissively of their schooling as having been largely irrelevant in preparing them for work. Those at university were generally more appreciative of their schooling. In other words, there appears to be something of a mismatch between what schools are offering and the perceptions young people have of their needs.

In another study[6], McCrindle outlines a 'sociological' classification of the current, identifiable generations of workers in Australian society — 'Baby Boomers' (born 1946-64); Generation X (born 1965-79); Generation Y (born after 1980). In relation to the domain of 'learning', he suggests that the characteristics of today's students are quite different from those of previous generations.:

	Baby Boomers	**Generation X**	**Generation Y**
Learning format	Formal Structured	Relaxed Interactive	Spontaneous Multi-sensory
Learning environment	Classroom style Quiet atmosphere	Round-table style Relaxed ambience	Café Style Music and multi-modal
Training focus	Technical Data/evidence	Practical Case studies/ applications	Spontaneous Multi-sensory

Table 1

At a purely pragmatic level we have also seen the recent extension of the school leaving age to at least 16 in all Australian jurisdictions. This means that we will now have more students staying longer at school. The community has an expectation that these young people will be provided with an appropriate mathematics program.

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[5] Saulwick, I. and Muller, D. (2007) *What Young People are Thinking,* Dusseldorp Skills Forum. Downloaded from http://www.dsf.org.au/papers/189.htm on 5 September 2007.

[6] McCrindle, M. 2006. *New Generations at Work: Attracting, Recruiting, Retraining and Training Generation Y*. McCrindle Research. Downloaded from http://www.mccrindle.com.au/wp_pdf/NewGenerationsAtWork.pdf on 5 September 2007.

In summary, school mathematics needs to respond to a client group — as students *and* as teachers — that is markedly different from those who have come before, and more diverse within itself.

Mathematics (the discipline) has changed in the last 30 years — implications for school mathematics

Note: This section is unfinished and consists of notes of conversations with a couple of significant Australian mathematicians to give some flavour. If the approach is appropriate we can seek the views of a range of mathematicians.

Preliminary feedback has suggested this section needs to be reworked significantly to make it much more reader friendly; and be much less daunting to the range of readers of the paper.

Large changes in mathematics over the past 30 years have been stimulated both by intra- and extra-mathematical advances. For example global analysis on the one hand and dynamical systems/instability on the other (just consider the effort now devoted to modelling environmental systems at both the global and local levels).

A nice example of a blending of the two influences is in the area of primality testing, where a desire for a really efficient algorithm has produced pretty good tests for primality but despite the pressure from cryptology we do not have comparable factorisation algorithms, in the sense that even the best computers likely to be produced prior to quantum technology will still require unusable time to factorise even only reasonably large numbers.

It is likely that it is in the area of sophisticated design of complex objects that technological change continues to affect every stage of the process from design to rolling off the assembly line, both by automating standard design and production methods to facilitating safe design at state-of-the-art levels.

In terms of doing the mathematics involved in all of this, the best CASs are used daily — witness the worldwide use of John Cannon's MAGMA package, for example.[7]

Some thoughts[8]:

Computers and their effects on mathematics:

- statistics - new field of computational statistics; led to real commercial statistics
- electronic communication - codes, ciphers, etc
- algebra – computers allow us to obtain new answers to new questions

Biological revolution - mathematics underpins a lot of this.

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[7] Notes of email conversation with Prof John Mack OAM.

[8] Notes of email conversation with Prof Cheryl Prager.

