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LINKING IT ALL TOGETHER

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This paper is a summary of the opening keynote at the AAMT conference 2015. It brings together various connections within mathematics through considering the use of rich tasks.

Introduction

Thank you so much for inviting me. I am delighted to be in Australia again, and in Adelaide, and I am greatly looking forward to conversations over the next few days about the work that you are doing in mathematics education.

It was over a year ago, at the BCME conference in Nottingham, when Will invited me. At that time I was the Director of NRICH[14]—a project which I know many of you already use. Since then I have changed job so the focus of my talk today will be slightly different from the title and abstract that I originally sent. In this plenary then I would like to combine some of what I had intended to talk about, with some new thoughts that have come to me as a result of my changed role.

Learners' views

Before I do that I am going to show you some short excerpts of English and Scottish children, mostly primary learners but not exclusively, talking about their experiences of mathematics. These film clips originated in a research project I did where I was interested in finding out what came to mind when learners thought about their mathematics lessons. I asked them several other questions too, which I will share with you later, but for now here are three or four short pieces for you to watch.

Whilst you are watching them, I would like you to consider two questions: firstly what are these children's views of mathematics, and secondly, if you asked your own students the same sort of question, would you get the same sort of responses or something different?

Mathematics to them is a set of mostly unconnected content. They talk about having 'done' fractions or decimals, for example. I put this somewhat down to the Numeracy Project in the UK, in which the year's plans were set out in unrelated chunks so learners

[14] NRICH: nrich.maths.org

might do some work on multiplication on Mondays and Tuesday and then swap to, say, identifying polygons for the rest of the week. Little attempt was made to tell any stories that connected the mathematics. Not only does this lead to a rather random view of the subject, we also know from the work of Brown and Askew (2001) that the most successful teachers look for and make connections in their teaching. So it is not surprising that the children in my study thought of mathematics as an unconnected set of topics. And mostly arithmetic, at that.

There were a handful who talked about using mathematics to solve problems, but again this was quite rare. Mostly mathematics is seen to be about learning to replicate or identify or calculate—although later when I did ask them if they knew any grownups who used mathematics, many said yes; teachers and people who go shopping! In the UK our assessment criteria use the words fluency and reasoning and these are often interpreted very narrowly by teachers.

The learners have a clear idea of how 'good' they are at the subject and know which ability group they fit. No Dweck growth mindset there! (Dweck 2006). But you will notice that they are not unhappy—they are quite satisfied with their lessons although in hundreds of hours of film, no-one ever got really excited when talking about their mathematics experiences! There is something here about expectations of just getting through what they have to do—if they are in year 4 there is a year 4 program to be got through and in a way that is their job as a student. I think, of everything, this is the bit that saddens me most. I suspect if I had asked them to tell me about almost any other subject—art, music, science, history—they would have been able to recall something creative that they did, something that was personal to them, perhaps creating a picture, playing a piece of music with others or dramatising a historical event. But hardly anyone talked about any personal mathematics they had done, discovered or been proud of. No-one thought of mathematics as a creative subject, or one in which they could have some personal input, whereas we know that the creativity of mathematicians has led to solutions of some of the world's problems.

Comparing curricula

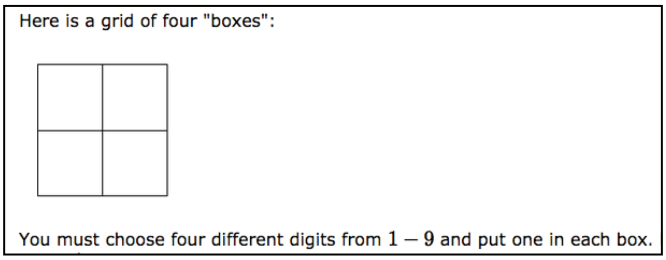
If we look at the Australian curriculum, we see that these ideas—creativity, enjoyment, confidence, connectedness, fluency, reasoning, problem posing and solving—are all embedded. Likewise in the UK, although we need two paragraphs to differentiate between aims and purpose. Nevertheless these ideas are thought important, and indeed when you read curriculum documents from across the world they all say much the same.

For me 'connectedness' is what mathematics is all about, though I would suggest that you are really lucky as a learner if you are let into that secret early on in your education. I am going to look at four ways in which mathematics can be thought of being connected, and these relate both to the curriculum aims and to our reactions to those film clips. And I hope you will be happy to do some mathematics along the way.

Making connections

Task 1: Reach 100

http://nrich.maths.org/1130



Make four two-digit numbers by reading top to bottom and left to right, then add them all up. Let us see who has made the largest sum? And the smallest?

I am hoping that this has provoked you into thinking about some questions you would like to ask. Here are some possibilities:

- What is the highest possible sum (and how do you know)?
- What is the lowest possible sum (and how do you know)?
- Can you make all the ones in between (what are your strategies and if they are not all possible why not?)
- Can you find a strategy for finding any sum? (e.g., work backwards?)
- What would happen of you changed the size of the grid?

Firstly the connection between fluency and reasoning. I am not sure how this is interpreted here, but in the UK it is very often taken to distinguish between algorithmic and factual recall speed on the one hand, and developing or following arguments on the other. My take on this is that you can and should promote them simultaneously. If we consider what knowledge and processes were need to get going on this task, we can see that here is an activity that supports fluency in the most restricted sense of the word, in that there is a lot of addition of two digit numbers involved. However to answer any of the subsequent questions there will, in addition to some trial and error, be some reasoning about place value and the relative importance of the position of the digits.

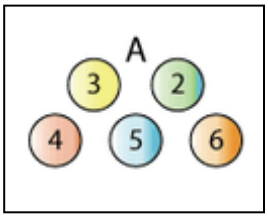
The phrase 'low threshold high ceiling' originated with Seymour Papert (1980). It means something that is accessible to all but with inbuilt challenge, and he used it to describe Logo. The phrase has been adopted by NRICH because it nicely describes these tasks which are suitable for a whole range of confidence and competence. Such rich tasks promote both fluency and reasoning, and make connections across domains, in this case number and algebra (though some would say that they are the same domain and that algebra is the structure of arithmetic). More than that (and to go back to the film clips), activities such as these, where learners are encouraged to pose their own problems, support confidence and creativity. They really are problems where you do not know what to do and there is no predetermined route to follow.

One way of using rich tasks is to let the learner just play around for a while in the environment, to get the feel of it, then offer a closed task which forces him/her to look at the structure of the environment—in this case the place value element. Then follow up with some multistep opportunities in which the learner has choices to make. Ruthven (1989) suggests that this exploration, codification and consolidation route is more profitable and fits better into a constructivist framework that the usual show, tell and practice routine that happens so often in many of our classrooms. Of course from a teacher's point of view the codification part, where the teacher makes the learning explicit and formalises it, makes great demands on the teacher who has to react to what is happening in the classroom rather than deliver a preplanned lesson.

Let us have a look at another rich task, one that is aimed at an older audience but which I hope you will see has a wide range of appeal.

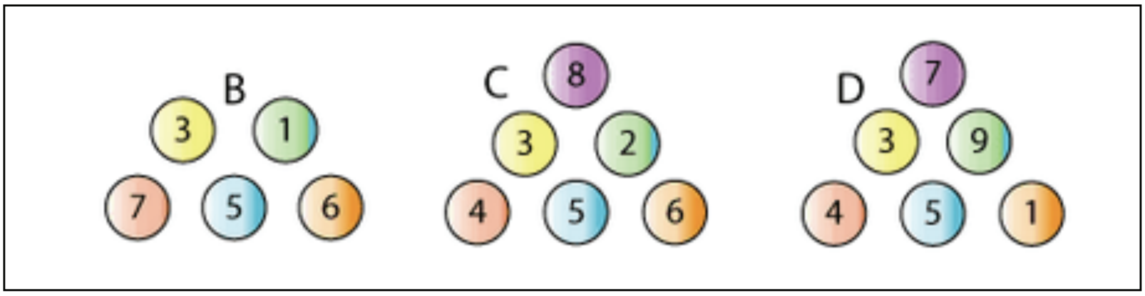
Task 2: Odds and evens

http://nrich.maths.org/4308
Here is a set of numbered balls. Assume I am playing against you.



I am trying to get an even sum and you are trying to get an odd sum. I go first and I pick two at random out of a bag—in this case 3 and 5. The sum is 8, this is even so I win a point. I replace the balls in the bag and you have a go. We keep going until we have each had, say, six goes. So my question is, is this a fair game and how do you know whether it is or it is not?

What if we chose a different selection of balls? Assuming we have to play using one of these sets, which is the fairest and how do you know?



I wonder how you worked this out. Perhaps you used a sample space, perhaps a tree diagram? Or perhaps you tried it out a few times and recorded your result? One of these is the fairest—but it is still not actually a fair game. Is it possible to choose a set of balls that would be totally fair? How would you go about it?

So here again we see the idea of playing around, being asked to do something specific which forces us to look at the structure of the given information, working up to a multistep problem. In each case the teacher's role is to help to formalise the learning and discoveries that have been made.

Rich tasks

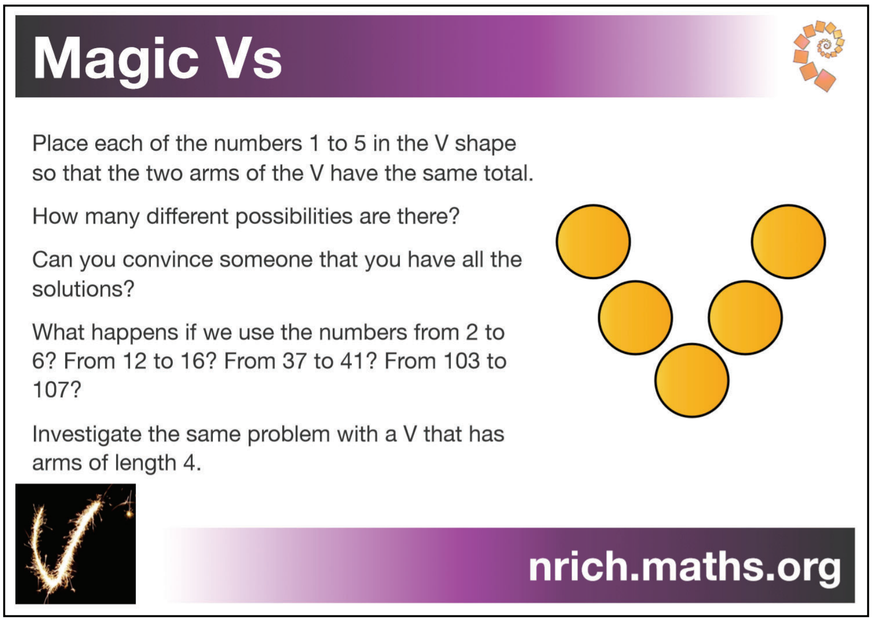
NRICH uses the term 'rich tasks' to describe activities such as these. Here are the characteristics of rich tasks, although not all tasks will exhibit all characteristics:

- combine fluency, problem solving and mathematical reasoning;
- are accessible, e.g., most students would be able to start (low threshold);
- promote success through supporting thinking at different levels of challenge (high ceiling);
- encourage collaboration and discussion (because talking often clarifies thoughts, and explaining and justifying are important parts of doing mathematics);
- use intriguing contexts or intriguing mathematics;
- allow for:
    - learners to pose their own problems,
    - different methods and different responses,
    - identification of elegant or efficient solutions,
    - creativity and imaginative application of knowledge;
- have the potential for revealing patterns or lead to generalisations or unexpected results;
- have the potential to reveal underlying principles or make connections between areas of mathematics.

Here are two other tasks which you may wish to try.

Task 3: Magic Vs

http://nrich.maths.org/6274



This problem gives opportunities for children to make conjectures, prove these conjectures and make generalisations. They will be practising addition and subtraction, and applying their knowledge of odd and even numbers.

Task 4: Simultaneous squares

The lines given by the following four equations enclose a square.

1. $y-2=x$
2. $y+x=6$
3. $y=x-1$
4. $y+x-3=0$

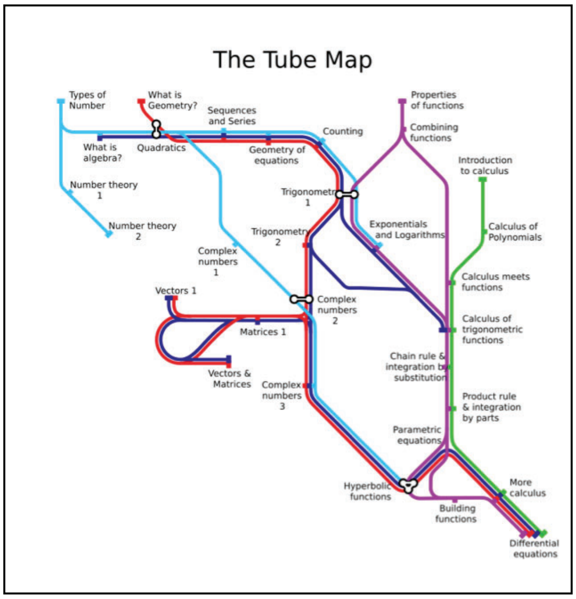
You might like to convince yourself of this before going any further!

- Given any three of the four lines that enclose a square, can you find the other one?
- You are given the area of the square and the coordinates of one vertex. Can you find possible equations of the four lines enclosing it?
- What is the minimum amount of information needed to be able to describe the equations of the four lines enclosing a square?

This problem provides an opportunity for students to draw together several different mathematical concepts that they have met previously, including the manipulation of linear equations, simultaneous equations, properties of straight-line graphs (including parallel and perpendicular lines), Pythagoras' Theorem, and the use of quadratic equations to solve geometrical problems.

Cambridge Maths

Both NRICH and my new project, Cambridge Maths, take all these connections very seriously. The Simultaneous Squares activity is taken from a new post-16 project called CMEP. The intention is to redesign the post-16 mathematics syllabus through making explicit the connections across and within mathematics. The big picture is based on the London Underground map and there are several possible routes through.



Cambridge Maths, of which CMEP will ultimately be a part, is a very ambitious project, funded by the University of Cambridge. The intention is to devise a mapping, or framework, of all the mathematics you might reasonably meet between the ages of 5 and 19. The framework will, as you might expect, make the most of the connections between the elements of mathematics. As well as the mapping there will be summative assessments at the end of various routes through the framework, a full set of resources to support those routes, and a comprehensive professional development offer to support teachers from novice to expert. The intention is that it would have international acceptance and in the long term would influence policy making, and especially in the UK! It is an amazing project to be part of and in the first instance I am working on the framework. I'm doing this through a web consultation and conversation and it would be great to have some input from Australian colleagues. You can follow the progress at www.cambridgemaths.org.

Task 5: Strike it Out

A final game. This is my all-time favourite. You need a 0–20 number line and someone to play with. Take turns to write an addition or subtraction calculation using three numbers from the line. Once you have written the calculation, cross the first two numbers off and circle the answer. The other player has to start their calculation with the circled number. So a set of responses could look like this:

3 + 4 = 7
7 – 5 = 2
2 + 15 = 17
17...

The winner is the player who goes last.

It is an engaging context for practising addition and subtraction, but it also requires some strategic thinking. It is easily adaptable, and can be used co-operatively or competitively.

Some questions to ask yourself:

- Is it possible to cross all the numbers off? Could you prove it?
- Is there a strategy for winning?
- What is the mathematical knowledge that is needed to play?
- Who would this game be for?
- What is the value added of playing the game?
- Could you adapt it to use it in your classroom?

In conclusion

Connectedness, creativity, confidence, fluency, problem solving—all these are possible through judicious choice of tasks. John Dewey summed it up beautifully:

"Give the pupils something to do, not something to learn; and if the doing is of such a nature as to demand thinking; learning naturally results."

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For more information

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