

Investigating the maths inside:

Stargazing with the SKA

Information for teachers



*Maths Inside* is a project funded by the Commonwealth Department of Education and Training under the Australian Maths and Science partnership Programme.

The aim of *Maths Inside* is to increase engagement of students in mathematics by using rich tasks that show the ways mathematics is used in real world applications.

# About this module

**This resource was developed by**

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This module consists of the video “The Square Kilometre Array” and the following activities:

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| --- | --- | --- |
| Activity 1 | Heavenly bodies | Years 7–9 |
| Activity 2 | (a) Big numbers | Years 7–9 |
|  | (b) Long distances | Years 8–10 |
|  | (c) Things that go really fast! | Years 8–10 |
| Activity 3 | (a) Playing with parabolas! | Years 9–11 |
|  | (b) Practical parabolas | Years 9–11 |
| Activity 4 | Seeking spirals | Years 8–10 |
|  | Spreadsheet A | Years 8–10 |
|  | Spreadsheet B | Years 8–10 |
| Activity 5 | Stars and parallax: Measuring the unmeasurable | Years 10–11 |

# Feedback

Feedback from teachers about these classroom activities would be appreciated. Please complete the form at <http://tiny.cc/mathsinsidefeedback>.

# Background

The Square Kilometre Array (SKA) is a multi-radio-telescope project that when complete will be the largest and most capable radio telescope available to scientists. It will allow scientists to study and collect information about the universe.

Radio telescopes detect the radio waves that are produced by physical occurrences in space, and then translate these waves into data and imagery which can be used by astronomers, often in conjunction with optical and other types of telescopes.

In its first phase, the SKA will be made up of three telescopes, each made up of thousands of small antennas—which will cover a total area of one square kilometre.

For more information, go to www.ska.gov.au.

Activity 1: Heavenly bodies

Students investigate the physical properties of different planets in the Solar System concentrating on using appropriate units. They then compare some of the properties (for example, mass) in raw figures, and then by using a unit base measure.

Students may also investigate the use of the Southern Cross in navigation.

# Why do this?

Looks at different measurements (for example, temperature, mass, time) and rates (for example, speed).

Makes comparisons between measurements, noting that some of the measurements will be extremely large, and some will be in scientific notation.

Simplifies comparisons by using a unit base measure, promoting proportional reasoning.

# Australian Curriculum links

Solve a range of problems involving rates and ratios, with and without digital technologies (ACMNA188)

# Getting started

A class should be able to name most, if not all, of the planets. If they need a prompt, you could use the mnemonic MVEMJSUN(P). Remember that Pluto is now considered a ‘dwarf planet’ and is not part of the Solar System.

Knowledge of the conditions on the planets may well be quite limited. Brainstorm what students might need to know to decide whether they could live on a particular planet or not.

The class will need internet access. This could be a take-home exercise.

\*Students may not have encountered scientific notation before and so their ordering may well rely on the leading digits while ignoring the exponent. Observation of the differences in exponents can lead to an informal discussion of scientific notation.

# My favourite planet

There is a lot of information suggested on the worksheet. You might like to select just a few for your class to do or give them different versions so that you can have ‘double-dips’ of planets.

Some of the information may be already have been calculated at the source (e.g. the mass of the planet when compared to Earth). If so, you could ask students to confirm those calculations.

When comparing the raw results (for example, the mass) each student could write their answer on a sticky note. Then the class could put the values in order from smallest to largest, explaining their reasoning.

You might want to arrange the students into ‘solar system groups’ so that each group could do the ordering and then compare to other groups.

(It would be helpful to have the facts for Earth as this will not be one of the planets investigated by the students).

# Resources needed

Worksheet

Internet access

Paper and pencil

Ruler

Sticky notes

# Further ideas

You could have the students represent each planet and arrange themselves in order from the distance from the sun. Ask the students to space themselves proportionally. This would be a good estimation activity and then could be checked using a tape.

You will need a space of at least 40m in length to do this.

Activity2A:

How big is a square kilometre?

Students will construct a square metre and work out how many of these are needed to make a square kilometre. They will then use maps to determine the extent of a square kilometre in a familiar area.

There are also several questions requiring calculations of circular areas in the context of telescopes located in Australia.

# Why do this?

Helps develop a sense of the size of a square metre and a square kilometre.

Uses formulas to calculate circular areas.

# Australian Curriculum links

Choose appropriate units of measurement for area and [volume](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=Volume) and convert from one unit to another [(ACMMG195)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMMG195)

Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area [(ACMMG197)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMMG197)

# Getting started

Ask students to nominate a familiar object with the approximate area of a square metre. \*This is a good opportunity to establish that just because area is measured in square units, this does not necessarily mean that the object is square.

How might we check this?

If a square metre is one metre by one metre, what could the dimensions of a square kilometre be?

Estimate a place/object that is a kilometre away. How might we check this?

## Making a square metre

The class could use some of the newspaper square metres to check the area of the objects they nominated. Try to find some other objects (maybe even a circular one) that have an area of one square metre.

# Resources needed

4 x one-metre rulers

Newspapers

Sticky tape

## A square kilometre around you

This is a good opportunity to revisit scales. Students can informally approximate the scale by relating to the length of a familiar object or building (such as the school hall) or calculate the scale more accurately.

# Resources needed

Internet access

Printer

Ruler and pencil

## Telescope 1 and 2

# Answers

1. 3217m2 (to nearest m2)

2. 311 (with a bit left over)

3. 3848m2 (to nearest m2)

4. Area increases by 19.6% whereas the diameter increases by 9%

Activity 2B: Very large numbers

Students complete a table to establish the pattern of expressing very large numbers in scientific notation. Relevant examples are used.

# Background

Large amounts of money are often quoted in billions or even trillions of dollars. Scientists and mathematicians often need to use very large numbers which they express in scientific notation.

# Why do this?

Reinforces the concept of powers of ten.

Indicates the sensible constructs that mathematicians use to communicate efficiently.

# Australian Curriculum links

Express numbers in scientific notation (ACMNA210)

# Getting started

Discuss very large numbers. What is the largest number that students know? (You might mention the googolplex).

What sort of things might be counted in very large numbers? Students may mention populations (human and animal), money, distances etc. If students have completed the first activity, My Favourite Planet, they will already have been exposed to very large numbers.

# Very large numbers

Try writing a very large number in “long-hand” on the board with a student timing you. Then write it in scientific notation, again being timed.

# Resources needed

Worksheet

Activity 2C: Things that go very fast

## Paperfolding to the moon

Students investigate doubling as the basis for counting data quantities. They produce a spreadsheet which shows the rapidity of increase when doubling.

# Background

The Square Kilometre Array will collect vast amounts of data. It is estimated that the data collected by the SKA in a single day would take nearly two million years to playback on an iPod.

# Why do this?

Demonstrates a simple exponential function.

Encourages the logical input of formulas into a spreadsheet.

Shows the efficiency of using a spreadsheet to do repetitive calculations.

# Australian Curriculum links

Investigate very small and very large time scales and intervals (ACMMG219)

# Getting started

How big is a gigabyte? Students can get a sense of data capacity by comparing the capacities of familiar technologies.

\*Students’ estimations for the number of folds needed to reach the moon are likely to be very much higher than the actual result. They may need convincing that the answer is indeed correct.

# Resources needed

Internet access

Ream of paper

Ruler

Excel spreadsheet application

# Further ideas

There are classic problems that use the concept of doubling. Some students might be interested in the Rice on the Chessboard problem.

## Things that go really fast

Students conduct an experiment using the different speeds of light and sound.

There are some follow-up questions on speed.

# Getting started

How fast is the Earth moving? This could be considered from the point of the speed on the surface of the Earth or its speed through space.

# Resources

Starting pistol and ear protection (alternatively use a torch and whistle)

Trundle wheel

Stopwatch

Activity 3A: Playing around with parabolas

Students fold paper to produce parabolic curves and compare the characteristics of the different parabolas. This leads to a more formal analysis where an equation for the curve of the Sydney Harbour Bridge is calculated.

# Background

The telescope dishes in the SKA are parabolic.

Parabolic reflectors collect energy (light, sound, radio waves) from a distant source and bring it to a common focal point.

# Why do this?

Gives students a hands-on way of creating a parabola.

Explores a variety of parabolic shapes and their characteristics leading to more formal analysis.

# Australian Curriculum links

Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate (ACMNA239)

# Getting started

What shape is a parabola? It may be helpful to have some images of parabolas and the way they are used in various objects.

# Folding a parabola

The folding instructions can be difficult to interpret. It is recommended that you do an example yourself first.

# Resources needed

Sheets of coloured A4 paper (at least two for each student)

Markers

# Further ideas

The YouTube references below outline two other alternative approaches where paper folding produces the outline of a parabola.

<https://www.youtube.com/watch?v=wtk5q8wGAe0>

<https://www.youtube.com/watch?v=vaLQawKuq8M>

Activity 3B: Practical parabolas

Students create a parabolic trough which can boil water (or cook a sausage).

Students create a parabolic paper dish to improve the sound on a mobile phone.

Further explorations are suggested.

# Why do this?

Demonstrates physically the effect of the focus/focal axis on light and/or sound.

Provides challenges in the actual construction of the trough leading to opportunities for practical problem-solving.

# Australian Curriculum links

Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate (ACMNA239)

# Getting started

What are parabolas good for? It may be helpful to have some images of parabolas and the way they are used in various objects.

# Practical parabolas

Students should work in groups of between two and four. There is a substantial amount of problem-solving involved in constructing the trough which may provide challenges for some groups.

# Resources needed

Reflective cardboard sheets (one per group)

Cardboard boxes, any size (one per group)

Scissors or box cutters

Sticky tape or masking tape

Rulers

Markers

Activity 4: Seeking spirals

Students investigate Archimedean and logarithmic spirals. They then draw both types of spiral by hand, before using technology to draw and change a logarithmic spiral.

Students simulate selecting random points to reduce redundancy firstly by hand, and then using spreadsheets.

# Background

This module develops a process to ensure that there are limited chances of redundancy, when telescopes are placed as part of the grid in the Square Kilometre Array.

The SKA scientists are attempting to find a way of reducing the number of telescopes that are placed the same distance apart. The data provided by using two sets of telescopes that are the same distance apart will be identical and will not add to the amount of data available. They are anticipating placing the telescopes on a spiral rather than simply looking for a random pattern.

This webpage provides information about the arrangment of the telescopes in the Square Kilometre Array: <https://www.skatelescope.org/layout/>

# Why do this?

Introduces the concept of spirals, their important place in nature and their appearance in a range of situations.

Enables students to produce two types of spirals in different ways.

Uses spreadsheets to calculate the distances between many points to identify repeated values.

# Australian Curriculum links

Find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software [(ACMNA214)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMNA214)

Apply trigonometry to solve right-angled triangle problems (ACMMG224)

# Getting started

Why spirals? Vi Hart, a young ‘recreational mathemusician’, has created a series of three fascinating and visually stunning short videos that show you where to look for spirals: <https://www.youtube.com/watch?v=ahXIMUkSXX0>.

[Add a starred item(s) if necessary to help teachers identify some of the mathematical key points, and potential student misunderstandings]

## Finding spirals

This could be a take-home exercise.

## Drawing spirals

Students may need to have some explanation on polar graph paper and how to plot points.

# Answers

## Drawing spirals 1

The graph paper consists of concentric circles and lines radiating out from the centre at 10° intervals.

This is an Archimedean spiral, as the values are equally spaced. This can be determined by looking at the values in the Distance row, where the values go up by one for each 30°.

To determine the nature of the spiral from the graph requires many more points to be plotted.

## Drawing spirals 2

This is a logarithmic spiral, as the distances do not increase at a constant rate (as they do in an Archimedean one).

Random arrangement by hand

Half of the table is shaded as you do not need to measure *AB* then *BA*.

Random arrangement using Spreadsheet A



Spiral arrangement using Spreadsheet B



# Resources needed

Polar graph paper with a grid (You can download polar graph paper free: <https://incompetech.com/graphpaper/polar/)>

Pencil

Computer access

## Random arrangements

In **Random arrangement by hand** students first plot five points and measure the distance between all of the combinations of pairs of points. It may be easier for students to measure to the nearest centimetre. Sharing the results with the rest of the class *may* show that attempting to plot the points randomly does not always lead to different distances.

Using the computer to randomly select values would appear likely to solve the problem. In **Random arrangement using Spreadsheet A**, students are able to quickly generate 50 sets of 10 points by using the ‘refresh’ function. The distance is determined by using the formula for the distance between two points (*x*1, *y*1) and (*x*2, *y*2); i.e.



The results are surprising and indicate that selecting random points is not the answer.

This spreadsheet could be generated by the students themselves.

**Spiral arrangement using Spreadsheet B** indicate that points on a spiral do produce better results even if the number of points is increased from ten to twenty and then to thirty. This spreadsheet also explores the means to convert polar coordinates to Cartesian coordinates.

This spreadsheet could be generated by the students themselves.

# Resources needed

Paper and pencil

Ruler

Excel application loaded with the spreadsheets

# Further ideas

There is a simple method for drawing a logarithmic spiral by hand, like this:



The method is explained here: <http://mathworld.wolfram.com/LogarithmicSpiral.html>

Activity 5: Stars and parallax

Students calculate the distances to various stars using right angled trigonometry and parallax angles. They use astronomers’ measures such as the Astronomical Unit, parsecs and light years.

# Background

Powerful telescopes measure the position of a star, and the change in angle six months later. The angles measured are very small (1" or smaller).

Despite the fact that the distance between the Earth and the Sun appears to be a large number (1.50  108), the distances to the stars are much larger.

# Why do this?

Shows students that right-angled trigonometry is used in a realistic context, using extremely small angles and very large distances.

Indicates the sensible constructs that mathematicians use to communicate efficiently.

# Australian Curriculum links

Apply trigonometry to solve right-angled triangle problems (ACMMG224)

Express numbers in [scientific notation](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=Scientific+notation) [(ACMNA210)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMNA210)

Solve right-angled triangle problems including those involving direction and [angles of elevation and depression](http://www.australiancurriculum.edu.au/glossary/popup?a=M&t=Angles+of+elevation+and+depression) [(ACMMG245)](http://www.australiancurriculum.edu.au/curriculum/contentdescription/ACMMG245)

# Getting started

How far to the nearest star?

How far is a light year?

## Using parallax

This extends the exploration of the use of right-angled geometry into the world of astronomy. The calculations produce some very large numbers for the distance to nearby stars.

Students need to be careful that the angle they are using is the parallax angle and not the angle which is measured six months apart. The parallax is **half** of the angle measured six months apart. The first question shows that the calculated distance of a star from Earth or a star from the Sun are the same; a consequence of the very small angles.



**Angle measures**

1 revolution = 360o (degrees)

1° (degree) = 60' (minutes)

1' (minute) = 60" (seconds)

1c (radians) =  (degrees)

# Answers

1. 



The distance between the Sun and the star



The distance between the Earth and the star



2. Parallax angle =  = 0.75"



3. 

## Astronomers’ measures

Astronomers have introduced their own units of measurement (parsecs, AUs, light years). These units mean there is no need to use very large numbers. They highlight the precision of the measurements made with powerful telescopes (e.g. angles) and the enormous distances between objects in space. Attempting to draw scale drawings would further emphasise the difference in magnitude between the distances of the world we live in and the universe.

For parallax measurements made from Earth:

The nearest star is some 4.2 light years away, and that distance is known to within an accuracy of better than 0.1 light year.

For a star measured as 20 light years away, the distance estimate is accurate to within a light year or so.

For stars 50 light years away, the margin of error is perhaps 5 to 10 light years.

Beyond 100 light years, the parallax is so tiny it cannot be measured accurately, and astronomers resort to more indirect methods to find the distances to the stars. These relate to measuring the brightness of the star.

# Answers

4. 1 parsec = 3.26 light years = 206264.8 AU = 3.086  1013 km.

5. 

Check: 

6. What is the distance to 61 Cygni based on Bessel's measurement?

Distance  = 3.18 parsecs

7. [Proxima Centauri](https://en.wikipedia.org/wiki/Proxima_Centauri), the nearest star to Earth other than the Sun, has a parallax of 0.7687 arcseconds.

This calculates to  = 1.3009 parsecs (4.243 light years) distant

From question 2: 4.07  1013 ÷ (3.086  1013) = 1.3 parsecs

8. (a) Distance  parsecs

(b) The smallest angle 0.001" gives the greatest distance

(c) Distance  parsecs

(d) Distance  parsecs

(e) Wikipedia states: “[Spectroscopic](https://en.wikipedia.org/wiki/Spectroscopy) estimates of Rigel's distance place its distance between 700 and 900 [light-years](https://en.wikipedia.org/wiki/Light-year) (210 and 280 [parsecs](https://en.wikipedia.org/wiki/Parsec)). [*Hipparcos*](https://en.wikipedia.org/wiki/Hipparcos)'s 2007 measurement of its parallax gives a distance of 860 light-years (260 parsecs), with a [margin of error](https://en.wikipedia.org/wiki/Margin_of_error) of about 9% (80 light-years)” (https://en.wikipedia.org/wiki/Rigel)

The distance to Rigel has a margin of error suggesting any measure is only approximate and not stated with certainty.

9. Distance 

10. The diameter of Earth is 12 756 kilometres so the greatest distance between two points on the Earth is about 12 800 kilometres.

The distance between the Earth’s position in January and its position in July (6 months later) is 300 000 000 kilometres.

The parallax angles when the Earth has moved 300 000 000 km are extremely small, being measured in arcseconds which are th of a degree or 0.00028°.

Any angle measured between two points on Earth would be so small that it would be immeasurable.

### Challenge

The distance from a star to the sun is



where *b* is the distance between the Earth and the Sun in kilometres, and *P* the angles in degrees.

Generally, the parallax angle is very small and measured in seconds, therefore



As a hint we have been told that tan *P* = *P* when *P* is very small in radians. Then we need to convert  to radians.

Therefore



Astronomers use parsecs as a measure of distance*,* 1 parsec = .

But



Therefore



Therefore, to convert *d* to parsecs



Therefore parsecs when *P* is given in seconds.